A Parsimonious Approach to Forecasting the Yield Curve: The CIR# Model

Summary of the Thesis by Giuseppe Orlando

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This Thesis is about interest rates forecasting which is introduced by an explanation of the basics concepts on interest rates characteristics and determinants. Then, after a review of existing tools developed so far in the literature, the CIR# [16, 17, 18, 19, 15] is introduced.

The idea to work on interest rates originates from our experience in finance. On the verge of the financial crisis and in the ensuing years, we were struck by the number of financial companies that still relied on CIR model developed by Cox, Ingersoll & Ross in far 1985 [7]. While the model (contrarily to many others developed in literature) is parsimonious, understandable and manageable, on other hand it is indubitably outdated and, by construction, prevents negative interest rates.

Then, the challenge for us was to deal with the additional limitations inherent to the CIR model such as the absence of jumps, volatility dumping when interest rates are low, linear risk premium, etc. This has to be done by preserving the market volatility structure as well as the analytical tractability of the original CIR model.

Goal, original contribution and hypotheses

My original contribution to the specific literature is twofold: enhance the CIR model and turn it from a short-rate model into a forecasting tool for any yield curve.

The goal is to provide a new accessible methodology to forecast future interest rates which is quite powerful for the following reasons. First, all the improvements are obtained within the CIR framework in order to preserve the single-factor simplicity and the analytical tractability of the original model.

This is achieved by a suitable partition of data where we calibrate the CIR parameters by replacing the Wiener process in the random term of the CIR model with normally distributed standardized residuals. Those are the proceeds of an "optimal" ARIMA model suitably chosen in our procedure aimed at ensuring that the assumptions on which the CIR model is built are fulfilled. That allows capturing all the statistically significant time changes in the volatility of interest rates, thus giving an account of jumps and related market dynamics.

For the reason that the illustrated methodology is not a complete departure from the CIR model but is an enhancement, we have called it CIR#.

The scientific hypothesis is that the CIR# is outperforming other single factor models and that the new approach proves particularly useful in describing the term structure of interest rates post 2007 financial crisis. In fact, the model overcomes the inability of modelling negative/near-to-zero values and the issue of the built-in mechanism of dampening volatility when rates are low. Moreover, it provides an alternative to Monte Carlo for deriving the expected value of interest rates as the suggested discretization scheme returns the same results without the need of simulating 100,000 paths.

The idea to write a Thesis come to us during the visiting period in the academic year 2018-2019 at the Business Department of Financial Investments and Risk Management of the Wroclaw University of Economics and Business, directed by Prof. Krzysztof Jajuga. As the Thesis has been discussed and outlined in that period, all material published on interest rates forecasting from that time onwards, and particularly on the CIR#, is original and is the result of the advancements performed within the doctoral program.

1.1 The CIR# model

Cox, Ingersoll & Ross (1985)[7] proposed a stochastic model introduced by Feller (1951)[8] as follow

$$dr(t) = k(\theta - r(t))\,dt + \sigma\sqrt{r(t)}dW(t), \tag{1.1}$$

with initial condition $r(0) = r_0 > 0$. $(W(t))_{t\geq 0}$ denotes a standard Brownian motion under the measure $\mathbb{P}$ intended to model a random risk factor.

In the following, we will illustrate our original approach, but first let us recap what we believe to be the main issues that we want to address regarding the CIR model: **i.** Negative interest rates are precluded; **ii.** The diffusion term in CIR process goes to zero when $r(t)$ is small (in contrast with market data); **iii.** The instantaneous volatility σ is constant (in real life σ is calibrated continuously from market data); **iv.** There are no jumps (e.g. caused by government fiscal and monetary policies, by the release of corporate financial results, etc.).

To address the aforementioned issues, the first step is partitioning the available market data sample into sub-samples - not necessarily of the same size - in order to capture all the statistically significant changes of variance in real spot rates and consequently, to give an account of jumps. The second step consists in fitting an “optimal" ARIMA model to each sub-sample of market data and the third is to calibrate the parameters k, θ, σ by estimating them for each sub-sample of available data, as explained in Section 1.1.3.

1.1.1 STEP 1: ANOVA test and market data translation

The Analysis of Variance (ANOVA) is a parametric statistical tool developed by R.A. Fisher in 1918 as an extension of the t and the z test (both used for analysing statistical differences between two groups). The idea is to compare means (and relative variance) between three or more independent groups in a sample using the F-distribution. ANOVA tests the non-specific null hypothesis that the population's means are equal between groups (i.e. the "omnibus null hypothesis"). ANOVA is one-way (respectively two-way) when the number of independent variables is one (respectively two). The one-way ANOVA produces an F-statistic, obtained as a ratio of two estimates of population's variance. A higher ratio implies that the samples were drawn from populations with different mean values.

As explained in Section 1.1, our first objective is to overcome the issues pointed out in **i., ii.** and **iv.** for the CIR model. Thus we start to partition the whole data sample into sub-samples, which we call *groups*, by a one-way ANOVA analysis to highlight statistically significant changes of variance in market spot rates and so to give an account of possible jumps. The main difficulty concerns the choice of the optimal partition into groups to apply the ANOVA test; we had to take into account both the size (the smaller the group, the more refined the analysis) and the ability to capture any jumps (the larger the group, the better in terms of statistical significance).

1.1.2 STEP 2. Sub-optimal ARIMA models

The second step of our procedure consists in deriving the best fitting ARIMA(p, i, q) model to each group of interest rates partitioning the observed market data sample.

In time series analysis, an ARIMA (autoregressive integrated moving average) model is a generalization of an ARMA (autoregressive moving average) model. Fixed $p, q \in \mathbb{N}^*$, the ARIMA(p, q) refers to the model with p autoregressive terms and q moving-average terms, i.e. a given process $X(t)$ is ARIMA(p, q) if

$$X(t) = m + \sum_{h=1}^{p} \phi(h) X(t-h) + \varepsilon(t) + \sum_{h=1}^{q} \theta(h) \varepsilon(t-h),$$

where m is a constant, $\phi(h)$, $\theta(h)$ are the parameters of the model and $\varepsilon(t)$ is white noise.

1.1.3 STEP 3. Calibration of CIR parameters

Consider the jth-group partitioning the available market data sample, which we assume to be of length n_j. The calibration of the CIR parameters k, θ, σ in the group is performed as follows

- The volatility σ is estimated by the group standard deviation, namely $\hat{\sigma}_j$;
- The long-run mean parameter θ is estimated by the group mean, namely $\hat{\theta}_j$;
- The speed of mean reversion k is estimated by that value, say $\hat{k}_j$, solving the following minimization problem:

$$\min_{k>0} S_j(k) := \min_{k>0} \sqrt{\frac{\sum_{h=n_{j-1}+1}^{n_j} (u_h(k) - \overline{u}_j(k))^2}{n_j - 1}}. \tag{1.2}$$

For any $k > 0$, we define

$$u_h(k) := r_h(k) - r_{shift,h}, \quad h = n_{j-1} + 1, \cdots, n_j, \ (n_0 = 0) \tag{1.3}$$

being $r_{shift,h}$ the shifted market interest rate value in the jth-group, and $r_h(k)$ the corresponding simulated CIR interest rate value expressed as a function of the unknown parameter k. $\overline{u}_j(k)$ denotes the sample mean of $\{u_h(k),\ h = n_{j-1}+1, \cdots, n_j\}$. The $r_h(k)$ are calculated by applying the Milstein discretization scheme (1979)[13].

1.1.4 The change points detection problem

As explained in Section 1.1.1the main difficulty concerns the choice of the optimal segmentation to detect abrupt changes in the variance of the interest rates dynamics. In the literature there exist several approaches for detecting multiple changes in the probability distribution of a stochastic process or a time series such as sequential analysis (i.e., “online" methods), clustering based on maximum likelihood estimation (i.e. “offline" methods), minimax change detection, etc. (see, for example, Bai and

Perron (2003)[4], Lavielle (2005)[11] and (2006)[12], Hacker and Hatemi-J (2006)[10], Adams and MacKay (2007)[1], Arlot and Cenisse (2011)[3]).

1.2 Results

In this section firstly we are going to show the results over the dataset reported in the Thesis and, secondly, we will focus on turbulent periods where volatility is above the median.

Directionality of forecasting

In order to understand whether the forecast predicts correctly a rise or a drop of interest rates, we introduce the index of directionality (IDX). Let us denote r_t as the interest rate at time t and the corresponding forecast as r_t^f. We define the variable $\alpha_{t+1} := r_{t+1} - r_t$ as the difference between two consecutive interest rates, and the variable $\beta_{t+1} := r_{t+1}^f - r_t$ as the difference between the forecast at time $t+1$ and the actual interest rates at time t. Further, we consider the indicator variable $H(t+1)$ assuming only the values 0, 1 as follows

$$\begin{cases} H(t+1) = 1 & \text{if } sgn(\alpha_{t+1}) = sgn(\beta_{t+1}) \\ H(t+1) = 0 & \text{if } sgn(\alpha_{t+1}) \neq sgn(\beta_{t+1}). \end{cases}$$

We attribute the term of forecast “success" (in sign) when $H(t+1) = 1$. Therefore the index IDX is defined as an average of the $H(t+1)$ values on the number of forecasts over a time series of length T that is,

$$\mathrm{IDX} = \frac{1}{T-1}\sum_{t=1}^{T-1} H(t+1). \tag{1.4}$$

IDX indicates the percentage of correct predictions of interest rate directionality.

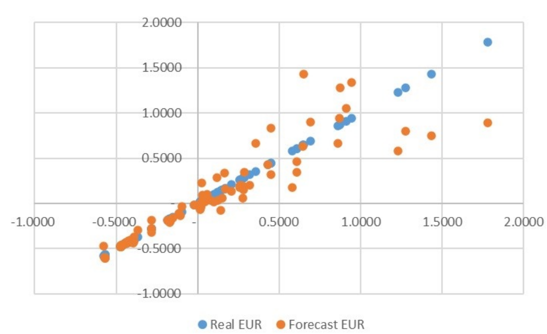
1.2.1 Forecasting results over the whole dataset

Let us start by plotting the performances versus the forecasts for all the four considered currencies. In Figure 1-1 we show that the forecasts closely follow the occurrences and they are not spread out.

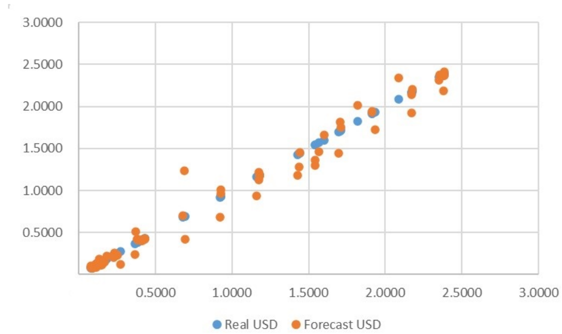
We supplement this visual analysis by considering the Bland–Altman plot (also called difference plot) which is a very popular tool in medicine and chemistry to analyse the agreement between two different methods [2], [6]. What inspired Bland and Altman was the need to know how much the outcomes of a method differ from another to be considered equivalent [1]. In our case, as the real data overlap with forecasts, we want to investigate the said agreement more clearly. Figure 1-2 [20] shows the said agreement between real data and forecasts. Based on this analysis it is possible to confirm the good agreement between data and forecasts as the maximum number of outliers is 0.885% (=9/113).

After having concluded the graphical analysis, we introduce the results based on the more traditional analysis consisting of the results obtained with the Normalized

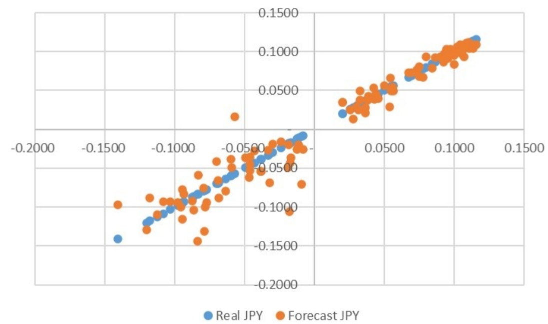
[1] In this instance we prefer the Bland-Altman plot to the well-known Tukey mean difference for a number of reasons: **a**) The first is based on the data while the second is based on its quantiles; **b**) In our case we have paired data as assumed by the Bland-Altman plot, the Tukey mean difference plot, instead, applies to either paired or unpaired data; **c**) While the Tukey mean difference plot answers the question of whether the variables have the same underlying distribution, the Bland-Altman answers the question on the differences between the pairs (which is more relevant as we want to know whether there is a systematic bias between real data and CIR# forecasts)



(a) EUR overnight. Plot of the realized interest rate occurrence versus the forecasted value.



(b) USD overnight. Plot of the realized interest rate occurrence versus the forecasted value.



(c) JPY overnight. Plot of the realized interest rate occurrence versus the forecasted value.

(d) CHF overnight. Plot of the realized interest rate occurrence versus the forecasted value.

Figure 1-1: Multiple comparisons for the overnight interest rate occurrences across currencies versus their corresponding forecasts.

Root Mean Square Error (NRMSE). In Tables 1.2, 1.3, 1.4 we compare the forecasting error (NRMSE) and the directionality of forecasting (IDX) for the EWMA, CIR_{adj}, CIR#. We have included the EWMA because it is a basic version of the Autoregressive Conditional Heteroscedasticity (ARCH) model, which is a common tool for forecasting time-varying financial data and a simple benchmark where no structure in data is assumed. The results show that, generally, the CIR# performs better over the whole dataset.

Table 1.1: Averaged NRMSE and IDX

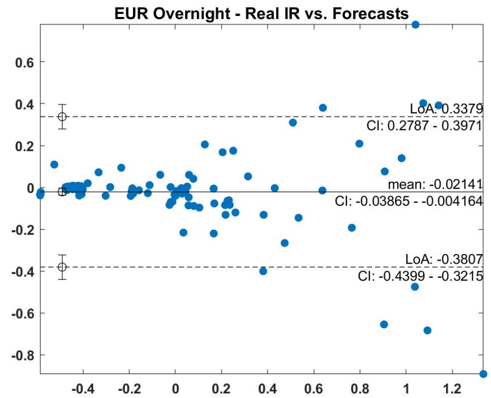
	EUR			USD			JPY		
	CIR#	**CIR$_{adj}$**	**EWMA**	**CIR#**	**CIR$_{adj}$**	**EWMA**	**CIR#**	**CIR$_{adj}$**	**EWMA**
NRMSE	3.47%	5.57%	11.41%	9.15%	13.60%	14.86%	5.31%	10.56%	9.41%
IDX	71.11%	65.60%	28.85%	62.77%	63.93%	38.55%	75.62%	70.39%	41.54%

Table 1.2: Averaged NRMSE and IDX

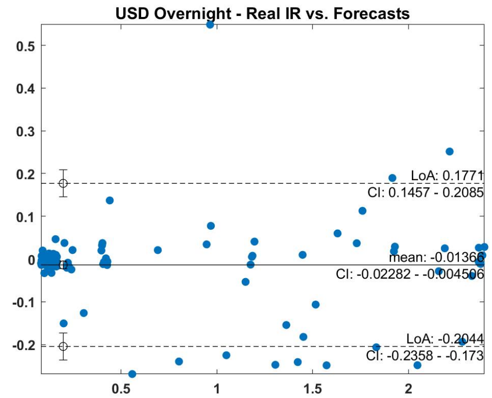
	EUR			USD			JPY			CHF		
	CIR#	**CIR$_{adj}$**	**EWMA**	**CIR#**	**CIR$_{adj}$**	**EWMA**	**CIR#**	**CIR$_{adj}$**	**EWMA**	**CIR#**	**CIR$_{adj}$**	**EWMA**
	Averages											
NRMSE	3.47%	5.57%	11.41%	9.15%	13.60%	14.86%	5.31%	10.56%	9.41%	8.26%	24.06%	11.54%
IDX	71.11%	65.60%	28.85%	62.77%	63.93%	38.55%	75.62%	70.39%	41.54%	53.38%	63.12%	57.51%

Table 1.3: NRMSE for different models, tenors and currencies

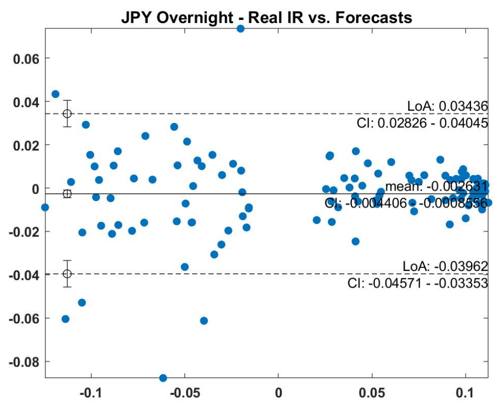
	EUR					
	EE00O/N	EE0001M	EE0002M	EE0003M	EE0006M	EE0012M
CIR #	5.14%	3.63%	3.56%	3.05%	2.71%	2.77%
CIR$_{adj}$	10.94%	5.39%	4.87%	4.39%	4.18%	3.64%
EWMA	11.54%	11.77%	11.61%	11.50%	11.11%	10.95%
	USD					
	US00O/N	US0001M	US0002M	US0003M	US0006M	US0012M
CIR#	7.88%	8.71%	8.53%	14.06%	7.83%	7.90%
CIR$_{adj}$	10.98%	28.15%	16.48%	10.98%	7.60%	7.75%
EWMA	15.69%	14.85%	15.07%	15.69%	14.32%	13.55%
	JPY					
	JY00S/N	JY0001M	JY0002M	JY0003M	JY0006M	JY0012M
CIR#	8.66%	5.20%	5.37%	4.85%	3.92%	3.88%
CIR$_{adj}$	13.55%	11.65%	9.26%	16.64%	6.82%	5.47%
EWMA	9.46%	9.82%	9.91%	9.60%	9.11%	8.56%
	CHF					
	CH00S/N	CH0001M	CH0002M	CH0003M	CH0006M	CH0012M
CIR#	9.29%	9.19%	8.53%	8.51%	7.51%	6.56%
CIR$_{adj}$	8.96%	16.80%	14.15%	12.59%	65.21%	26.73%
EWMA	12.47%	12.23%	12.11%	12.00%	10.95%	9.52%



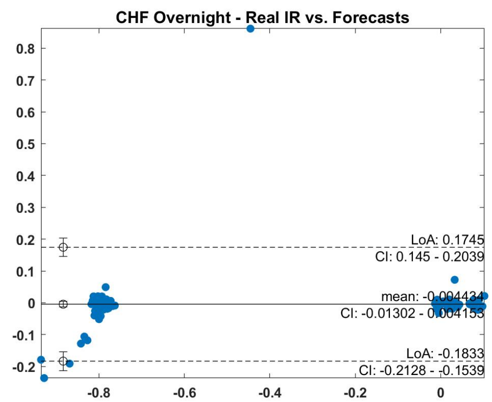
(a) EUR overnight. Over 113 data points outliers are 9 (out of which only 5 outside LoA confidence level).



(b) USD overnight. Over 113 data points outliers are 9 (out of which only 6 outside LoA confidence level).



(c) JPY overnight. Over 113 data points outliers are 5 (all of them outside LoA confidence level).



(d) CHF overnight. Over 113 data points outliers are 2 (none of them outside LoA confidence level).

Figure 1-2: Bland-Altman plot. Multiple comparisons for the overnight interest rates occurrences across currencies versus their corresponding forecasts. Maximum number of outliers are 0.885%.

1.2.2 Forecasting results in turbulent periods

As mentioned, through our procedure we can identify clusters of volatility. In Section 1.2.1 we have shown the averaged performance over the whole dataset and for each currency (Tables 1.3, 1.4). In this Section we illustrate how the CIR# model performs

Table 1.4: IDX for different models, tenors and currencies

	EUR					
	EE00O/N	EE0001M	EE0002M	EE0003M	EE0006M	EE0012M
CIR #	66.67%	71.67%	75.00%	75.00%	68.33%	70.00%
CIR$_{adj}$	53.73%	68.65%	65.67%	68.65%	65.67%	71.64%
EWMA	38.80%	28.35%	25.37%	29.85%	25.37%	25.37%
	USD					
	US00O/N	US0001M	US0002M	US0003M	US0006M	US0012M
CIR #	60.00%	58.33%	58.33%	66.67%	60.00%	73.33%
CIR$_{adj}$	73.13%	49.25%	50.74%	73.13%	71.64%	65.67%
EWMA	47.76%	40.29%	32.83%	47.76%	26.86%	35.82%
	JPY					
	JY00S/N	JY0001M	JY0002M	JY0003M	JY0006M	JY0012M
CIR #	67.16%	71.64%	74.62%	70.14%	86.56%	83.58%
CIR$_{adj}$	64.17%	77.61%	65.67%	65.67%	77.61%	71.64%
EWMA	56.71%	38.80%	47.76%	47.76%	26.86%	31.34%
	CHF					
	CH00S/N	CH0001M	JY0002M	CH0003M	CH0006M	CH0012M
CIR #	53.09%	53.09%	57.52%	52.21%	51.32%	53.09%
CIR$_{adj}$	69.02%	66.37%	61.06%	67.25%	59.29%	55.75%
EWMA	61.06%	56.63%	61.06%	61.06%	52.21%	53.09%

when volatility is high and forecasts are more challenging. This corresponds to considering the overnight (as it is the most exposed to market sentiment), which we have partitioned into clusters of volatility. Then, for each currency, we have selected the two clusters with higher volatility. Table 1.5 compares the forecasting error and the index of directionality for the EWMA, CIR$_{adj}$, CIR#. Note that the volatility ratio shows the volatility of the cluster over the median volatility recorded on the whole dataset for the selected currency. As displayed the CIR# performs better in any situation.

1.3 Testing and validation

In this Section, the basic question we want to answer is how our forecasts differ from original time series, purely random data and noise? For the reason of space, we show

Table 1.5: NRMSE and IDX in turbulent periods for the CIR#, CIR_{adj} and EWMA over overnight maturities and different currencies

	EUR		USD		JPY		CHF	
Cluster	1-13	39-52	3-8	28-33	49-54	62-68	12-19	29-60
Volatility ratio	$\frac{7.63\%}{10.00\%}$	$\frac{18.33\%}{10.00\%}$	$\frac{4.07\%}{1.26\%}$	$\frac{1.60\%}{1.26\%}$	$\frac{0.83\%}{0.49\%}$	$\frac{3.35\%}{0.49\%}$	$\frac{3.5\%}{1.22\%}$	$\frac{23.63\%}{1.22\%}$
NRMSE CIR#	72.70%	24.80%	27.90%	16.10%	50.00%	38.46%	54.31%	16.73%
NRMSE CIR_{adj}	72.70%	26.50%	43.50%	25.50%	67.10%	81.00%	67.46%	27.80%
NRMSE EWMA	54.50%	27.32%	61.40%	49.2%	56.30%	52.80%	98.12%	26.78%
IDX CIR#	36.30%	76.9%	100.00%	100.00%	80.00%	83.30%	66.67%	66.57%
IDX CIR_{adj}	35.30%	53.84%	80.00%	80.00%	60.00%	83.30%	66.65%	64.28%
IDX EWMA	36.60%	30.70%	40.00%	20.00%	60.00%	66.66%	64.28%	51.42%

the analysis on the EUR Overnight but we got similar results for all considered time series.

1.3.1 Results on test data

This analysis is carried out with R on both real and fabricated data described in the Thesis and the purpose of creating that data is the following: is the analysis we intend to run valid and consistent? If yes, time series 1 (EUR1) a 5 (a copy of EUR1) should be identical while time series 1 and 2 (random) should be unrelated. After having passed that check, the next question is: does time series 6 (EUR1 forecast) look similar to time series 1 or it does resemble more 2 or 3 (EUR1 + noise)?

Apart from the classical instruments used for testing the goodness of fit, in this section we measure the distance between data with a heat map [22]. As a metric, we adopt a generalization of both the Euclidean and the Manhattan distance is the Minkowski distance [9], [5], [21]. As expected Figure 1-3 shows that the EUR1 times series and its copy are identical (which confirms that the analysis is able to correctly identify this feature). The forecasts follow immediately after. Noise and random are recognized as similar and they group with the EUR1 changed sign time series. Hence, we can conclude that our model both spatially and hierarchy is very close to reality and, so, fits well with the intended purposes.

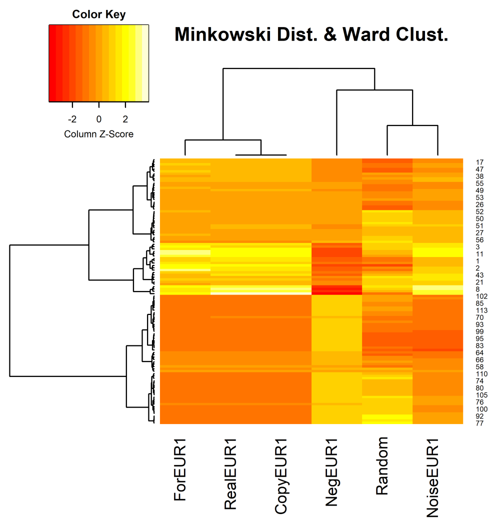


Figure 1-3: EUR overnight and test data. Minkowski distance and dendrogram based on Ward's criterion.

1.4 Conclusions

The goal of this Thesis is to provide a new accessible methodology to forecast future interest rates where all the improvements are obtained within the CIR framework in order to preserve the single-factor simplicity and the analytical tractability of the original model. The scientific hypothesis is that the so-called CIR# outperforms other single factor models and that the new approach proves particularly useful in describing the term structure of interest rates post 2007 financial crisis.

In this work, we have shown that the CIR# model, while preserving those features, is capable of coping with negative interest rates, cluster volatility and jumps.

This has been tested on money market interest rates during turmoil and calmer periods, by measuring the directionality of rates as well as the forecasting error. Besides that, we have shown how the results of the model could be tested and validated with several metrics and clustering criteria.

Apart from the scientific confirmations on both goal and hypothesis achieved during those years of work on the Thesis [17, 18, 19, 15], the proposed model has been adopted by a number of financial institutions. For example, VTB capital mentioned that "due to RUONIA's lack of an OIS term structure, it is impossible to build an arbitrage-free strategy to compare floaters with bullets. Using other curves, such as MosPrime, IRS or swaps on CBR's key rate, for forecasting the future RUONIA trajectory exposes the analyst to the basis with RUONIA, which has been historically unstable. Such approach also suffers from subjective assumptions, which makes the estimates of z-spreads poorly comparable". Instead, "an enhanced CIR model produces robust and reproducible RUONIA forecasts without any arbitrariness, giving common ground upon which to estimate spot and historical z-spreads" Naumov et al. (2021) [14].

In terms of implications for central banks, the ability to correctly model and predict interest rates for the effectiveness of inflation targeting is essential. While the adopted models are comprehensive and provide a full sketch of policy changes, with up to 1,000 equations they might be difficult to handle. In this respect, a single equation and risk factor model such as the CIR# could provide a quick answer when needed.

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